Pearson Edexcel

Mark Scheme(results)

January 2021

Pearson Edexcel International Advanced Level in Pure Mathematics P2
Paper WMA12 / 01

Question Number	Scheme	Marks
1(a)	$\mathrm{f}(-1)=(-1)^4+a(-1)^3-3(-1)^2+b(-1)+5=4$	M1
	$1-a-3-b+5=4 \Rightarrow a+b=-1$ *	A1*
		(2)
(b)	$\mathrm{f}(2)=(2)^4+a(2)^3-3(2)^2+b(2)+5=-23$	M1
	$\Rightarrow 8a+2b=-32$ oe (eg $4a+b=-16$)	A1
	$b=-1-a \Rightarrow 4a-1-a=-16 \Rightarrow a=...$	dM1
	$a=-5, b=4$	A1
		(4)
		(6 marks)

(a) **NB Do not apply a misread on the equation *a* + *b* = −1 as it gains no extra marks. They can score a maximum of M1A0 M1A1M1A0 if they use an incorrect equation such as *a* + b = 1.**

M1 Attempts to substitute ± 1 into f(x) and set equal to 4 to obtain an equation in *a* and *b*. Powers of −1 need not be seen, accept e.g. 1 – a – 3 – b + 5 = 4. Condone invisible brackets on powers for the M mark, but penalise in the A (if incorrect).
Alternatively, attempts long division reaching a remainder which is then set equal to 4. Look for a quotient starting $x^3+..$ and linear remainder in *a* and *b*. It may be seen in tabulated form.
Another alternative method: Writes $f(x)=(x+1)\left(x^3+\alpha x^2+\beta x+\gamma\right)+4$ (allow with γ = 1), expands and equates coefficients, and proceeds to eliminate α, β and γ from the equations.

A1* Rearranges the equation with no errors and achieves the given answer with at least one intermediate line with power evaluated. FYI by long division the quotient is $x^3+(a-1)x^2+(-a-2)x+a+b+2$ and the remainder is $3-a-b$.
By equating coefficients look for $\gamma+4=5$ (may be implied), $a=\alpha+1$, $b=\beta+1$, $-3=\alpha+\beta$ being solved correctly.

(b)

M1 Attempts to substitute ± 2 into f(x) and set equal to ± 23 to obtain an equation in *a* and *b*. Again, accept attempts at long division leading to a remainder linear in *a* and *b* (which is $8a+2b+9$), which is then set equal to ± 23, or via equating coefficients (look for at least three equations formed). May have substituted for *a* or *b* already.

A1 $8a+2b=-32$ oe need not be fully gathered but powers should be evaluated. So e.g. accept $16+8a-12+2b+5=-23$ or accept $16+8a-12+2"(-a-1)"+5=-23$ if substitution for *b* happens first (similar for *a*). Via equating coefficients, all equations should be correct.

dM1 Attempts to solve simultaneously and achieves a value for *a* or *b*. It is dependent on the previous method mark. Allow as long as values for *a* or *b* appears after writing out their two equations in both *a* and *b* (or solving a linear equation if substitution occurs before remainder theorem). Allow for slips in copying their initial equations.

A1 $a=-5, b=4$ cao

Question Number	Scheme	Marks
2(a)	$\left(\frac{dy}{dx}=\right)\ 3x^2-2x-16$	M1A1
	$3x^2-2x-16=0 \Rightarrow x=...$	M1
	$x=\frac{8}{3},-2$	A1
		(4)
(b)	$\left(\frac{d^2y}{dx^2}=\right)\ Cx+D$	M1
	$\Rightarrow \frac{d^2y}{dx^2}=C\times"..."+D(>0)\Rightarrow \text{min}\ \text{ or }\ (<0)\Rightarrow \text{max}$	dM1
	All three components: $\left(\frac{d^2y}{dx^2}=\right)\ 6x-2$ when $x=\frac{8}{3},\ \frac{d^2y}{dx^2}=...(=14)>0\Rightarrow \text{min}$ when $x=-2,\ \frac{d^2y}{dx^2}=...(=-14)<0\Rightarrow \text{max}$	A1
		(3)
Alt (b)	E.g. $x=-2.5,\frac{dy}{dx}=\frac{31}{4},x=0,\frac{dy}{dx}=-16$	M1
	E.g. $x=-2.5,\frac{dy}{dx}=\frac{31}{4}>0,x=0,\frac{dy}{dx}=-16<0$ so $x=-2$ is a maximum $\left(x=0,\frac{dy}{dx}=-16<0\right),x=3,\frac{dy}{dx}=5>0,$ so $x=\frac{8}{3}$ is a minimum	dM1 A1
		(3)
		(7 marks)

(a)

M1 Differentiates to achieve $\left(\frac{dy}{dx}=\right) Ax^2+Bx+C$ where $A,B,C\neq 0$

A1 $\left(\frac{dy}{dx}=\right) 3x^2-2x-16$

M1 Sets their $\frac{dy}{dx}=0$ and attempts to solve their quadratic. Usual rules apply. Setting equal to zero may be implied by an attempt to solve the quadratic.

A1 $x=\frac{8}{3},-2$ or equivalent single fraction (do not accept 2.67 but $2.\dot{6}$ can be accepted.)

(b)

M1 Attempts to differentiate their $\frac{dy}{dx}$ to achieve $\left(\frac{d^2y}{dx^2}=\right)$ $Cx+D$ where $C,D\neq 0$. Allow if found in part (a)

dM1 Substitutes in one of their x values for part (a) into their $\frac{d^2y}{dx^2}$ and draws a consistent conclusion with their value found. Accept with correct unsimplified values **OR** *with correct reason*, e.g. $\frac{d^2y}{dx^2}<0$ hence maximum, if no substitution seen It is dependent on the previous method mark.
Accept "concave down" for a maximum and "concave up" for a minimum.

A1 Correct second derivative, correct values (need not be simplified but if simplified values seen they must be correct) for $\frac{d^2y}{dx^2}$ at $x=\frac{8}{3},-2$ and draws a correct conclusion with reason (>0 or <0, positive or negative) for both. It must be clear which point corresponds with each conclusion.
NB Allow M1dM1A0 if no substitution is seen as long as reason is given, e.g. When $x=-2$, $\frac{d^2y}{dx^2}<0$ hence max.

Alt (b)

M1 Attempts to evaluate $\frac{dy}{dx}$ both side of at least one of their stationary points.

dM1 Evaluates $\frac{dy}{dx}$ between their two stationary points and the other side of at least one of them, considers signs and draws a consistent conclusion.

A1 Evaluates $\frac{dy}{dx}$ correctly either side of both of their stationary points (three points required, you may need to check the evaluation) and draws correct conclusions for both.

A method via calculus is required, so reasoning from the shape of a cubic scores no marks.

Question Number	Scheme	Marks
3(i)	$7^{x+2}=3\Rightarrow 7^2\times 7^x=3$	M1
	$\Rightarrow x=\log_7\dfrac{3}{7^2}$	A1
	$x=\log_7\dfrac{3}{49}$	A1
		(3)
Alt(i)	$7^{x+2}=3$	
	$(x+2)\log_7 7=\log_7 3$	M1
	$\Rightarrow x=\log_7 3-2$	A1
	$x=\log_7\dfrac{3}{49}$	A1
		(3)
(ii)	$1+\log_2 y+\log_2(y+4)=\log_2(5-y)$	
	E.g. $1+\log_2 y(y+4)=\log_2(5-y)$ or $1\to\log_2 2$	M1
	$\log_2\left(\dfrac{y(y+4)}{5-y}\right)=-1\Rightarrow\dfrac{y(y+4)}{5-y}=\dfrac{1}{2}$ or $\log_2\left(2y(y+4)\right)=\log_2(5-y)\Rightarrow 2y(y+4)=5-y$	dM1
	$2y^2+9y-5=0$	A1
	$(2y-1)(y+5)=0\Rightarrow y=...$	ddM1
	$y=\dfrac{1}{2}$	A1
		(5)
		(8 marks)

(i) Note this appears as MMA on ePEN but is being marked MAA.

M1 Applies the addition law to write the equation in the form $7^{\cdots} \times 7^{x} = 3$.
Alternatively takes logs on both sides and applies the power law (condone invisible brackets). Allow any base for the logarithm.

A1 Any correct exact expression for *x*, such as $x = \log_7 \frac{3}{7^2}$, $\log_7 3 - \log_7 49$, $\log_7 3 - 2$, $\frac{\log 3}{\log 7} - 2$ etc. Allow with any base for the logarithm.

A1 $\log_7 \frac{3}{49}$ Correct answer only.

(ii)

M1 Correctly applies the addition or subtraction law of logs at least once to combine two terms at some stage in their working (may be on incorrect terms) **or** correctly replaces 1 with $\log_2 2$ at some point (oe).

dM1 Attempts to rearrange the equation to the form $\log_2(...) = C$ or $\log_2(...) = \log_2(...)$ **with no incorrect log** work and removes the logs correctly. (There may be slips when rearranging terms but all log work should be correct.) It is dependent on the previous method mark.
May see $\log_2 2y(y+4) = \log_2(5-y) \Rightarrow 2y(y+4) = 5-y$ (may be implied by a correct equation).

A1 $2y^2 + 9y - 5 = 0$ oe 3TQ

ddM1 Attempts to solve their quadratic. Usual rules apply. It is dependent on them achieving a 3TQ and both previous method marks.

A1 $y = \frac{1}{2}$ only (accept decimal equivalent). (The −5 must have been rejected if seen.)

SC Allow M1dM0A0 SC M1A1 (if the A is appropriate) for the solution

$$1 + \log_2 y + \log_2(y+4) = \log_2(5-y) \to \frac{\log_2 y(y+4)}{\log_2(5-y)} = -1 \to \log_2 \frac{y(y+4)}{5-y} = -1 \to \frac{y^2+4y}{5-y} = \frac{1}{2}$$

etc, allowing the recovery from the incorrect division of logs as a log for the last two marks.

Question Number	Scheme	Marks
4(a)	$(2+px)^6=(2^6+)\ 6\times 2^5(px)+\frac{6\times 5}{2}\times 2^4(px)^2+...$ or $\left(1+\frac{"p"}{2}x\right)^6=1+6\times\left(\frac{"p"}{2}x\right)+\frac{6\times 5}{2}\times\left(\frac{"p"}{2}x\right)^2+...$	M1
	2^6 or 64	B1
	$+192px$ or $+240p^2x^2$	A1
	$(64+)192px+240p^2x^2$	A1
		(4)
(b)	$\left(3-\frac{1}{2}x\right)(2+px)^6\Rightarrow\left(3-\frac{1}{2}x\right)(64+192px+240p^2x^2)$	
	Attempts $3\times"240p^2"$ and $\left(-\frac{1}{2}\right)\times"192p"$	M1
	$3\times"240p^2"+\left(-\frac{1}{2}\right)\times"192p"=-\frac{3}{4}$	dM1
	$2880p^2-384p+3=0\Rightarrow p=...$	ddM1
	$(p=)\ \frac{1}{8},\frac{1}{120}$	A1
		(4)
		(8 marks)

(a)

M1 Attempts to expand using the binomial theorem in an attempt to find term 2 and term 3. Look for correct binomial coefficients linked with correct powers of x, condoning invisible brackets and the p not being squared on the third term. Allow slips on powers of 2, or if 2^6 has been taken out. The binomial coefficient must have been evaluated but need not be simplified and at least one must be correct. Terms may be found separately.

B1 Sight of 2^6 or 64 as the first term or extracted from the bracket.

A1 $+192px$ or $+240p^2x^2$ having gained the M. Allow also for $2^6\left(1+3px+\frac{15}{4}p^2x^2\right)$ and accept with $(px)^2$

A1 $+192px$ and $+240p^2x^2$ correct x and x^2 terms, may be as part of a list. Accept with $(px)^2$ ISW after a correct solution if they go on to try and divide through.

(b)

M1 Identifies the two x^2 terms either separately or may be seen as part of a full expansion – look for two x^2 terms in their expansion. There may be slips in the coefficients but they should be coming from the correct combination of terms in the brackets.

dM1 A complete method to find the term in x^2 and set the coefficient equal to $-\frac{3}{4}$ to obtain an equation in p. The correct two terms in x^2 for their expansion in (a) must have been identified. It is dependent on the previous method mark.

ddM1 Attempts to solve a 3TQ in p. Usual rules apply for solving a quadratic but it is dependent on the previous two method marks

A1 $(p=)\ \frac{1}{8},\frac{1}{120}$ Accept decimal equivalents, 0.125 and awrt 0.00833

Question Number	Scheme	Marks
5(i)	(As $x \geqslant 0$ so $\sqrt{3x}$ exists and) $\left(\sqrt{3x}-1\right)^2 \geqslant 0$	M1
	Hence $3x-2\sqrt{3x}+1 \geqslant 0$	M1
	$\Rightarrow 3x+1 \geqslant 2\sqrt{3x}$ *	A1*
		(3)
Alt 1	$3x+1 \geqslant 2\sqrt{3x} \Leftrightarrow (3x+1)^2 \geqslant 12x \Leftrightarrow 9x^2-6x+1 \geqslant 0$	M1
	$9x^2-6x+1 \geqslant 0 \Leftrightarrow (3x-1)^2 \geqslant 0$	M1
	Square numbers are greater than or equal to zero so $(3x-1)^2 \geq 0$ is true hence $3x+1 \geqslant 2\sqrt{3x}$ *	A1*
		(3)
Alt 2	If $3x+1 < 2\sqrt{3x}$ then $3x-2\sqrt{3x}+1 < 0$	M1
	So $(...\sqrt{3x} \pm ...)^2 < 0$ or $(\sqrt{x} \pm ...)^2 < 0$	M1
	But $(\sqrt{3x}-1)^2 \geq 0$ for all $x \geqslant 0$ so $3x+1 \geqslant 2\sqrt{3x}$	A1
(ii)	Shows that it is not true for three consecutive prime numbers Eg $5+7+11=23$ which is not divisible by 5 (so not true)	B1
		(1)
		(4 marks)

(i)

M1 Uses that ($x \geqslant 0$ and) squares are non-negative to set up a suitable equation.

M1 Squares to achieve 3 terms.

A1 Rearranges correctly to the given result.

Alt 1: (Backwards proof)

M1 Starting with the given statement, attempts to square both sides, expand $(3x+1)^2$ (three terms required) and collect terms on one side of the inequality.

M1 Attempts to complete the square/factorise the expression to achieve a perfect square or (following error) $(..x+..)^2+..$ (inequality not needed here).
Alternatively, uses other valid method (such as discriminant is zero) to show the resulting expression is non-negative. Finding a single solution alone is not sufficient, there must be a reason why the expression is never negative. (E.g. for $9x^2-6x+1=0$ discriminant is $(-6)^2-4\times 9=0$, so single root hence as positive quadratic, $9x^2-6x+1 \geqslant 0$)

A1* Achieves $(3x-1)^2 \geq 0$ and a statement such as since this latter equation is true (as squares are never negative) hence $3x+1 \geq 2\sqrt{3x}$. Alternatively, they may achieve an inequality such as $(\sqrt{3x}-1)^2 \geq 0$ and conclude in a similar way.
Note that the proof should really have two way implications at each stage, but allow full credit for proofs that do not show this.

Alt 2: (Contradiction type proof)

M1 **Starts with the negation** of the statement and gathers terms on ones side of the equation.

M1 Attempts to factorise (oe method) the resulting expression to achieve a perfect square.

A1* Achieves $(\sqrt{3x}-1)^2 < 0$ or alternative suitable expression and concludes as this is false the starting assumption was false, hence the original statement is true.
NB They may substitute for $\sqrt{3x} = y$ or similar, which is fine and can score full marks if correctly reasoned.

NB: there are variations between the methods, but in general look for setting up a correct equation/gathering and identifying the underlying quadratic for the first M, attempting to factorise to a perfect square/complete the square to a non-negative expression or expand a perfect square (as appropriate) for the second M, and with all steps correct and suitable conclusion for the final A. Simply rearranging the equation alone is not sufficient for the first M.

(ii)
B1 Provides a correct counter example, such as 5 + 7 + 11 = 23, or 11 + 13 + 17 = 41, and gives a conclusion. Must see the sum evaluated correctly. The conclusion may be minimal e.g. “hence shown”, “not divisible by 5” etc.
The conclusion should come from a correct example, so if they conclude from an incorrect example, it will be B0. But if they have multiple examples, at least one of which is a correct one, and a generic conclusion, B1

NB 1+2+3 is **not** a valid counter example as 1 is not prime.

Question Number	Scheme	Marks
6(a)	$\dfrac{3\sin\theta\cos\theta}{2\sin\theta-1}=5\tan\theta \Rightarrow \dfrac{3\sin\theta\cos\theta}{2\sin\theta-1}=5\underline{\dfrac{\sin\theta}{\cos\theta}}$	M1
	$\dfrac{3\sin\theta\cos\theta}{2\sin\theta-1}=5\dfrac{\sin\theta}{\cos\theta} \Rightarrow 3\sin\theta\cos^2\theta=5\sin\theta\left(2\sin\theta-1\right)$	M1
	$3\sin\theta\cos^2\theta=10\sin^2\theta-5\sin\theta \Rightarrow 3\sin\theta(1-\sin^2\theta)=10\sin^2\theta-5\sin\theta$ *	M1
	$\Rightarrow 3\sin^3\theta+10\sin^2\theta-8\sin\theta=0$	A1*
		(4)
(b)	$3\sin^3 2x+10\sin^2 2x-8\sin 2x=0 \Rightarrow \sin 2x(3\sin^2 2x+10\sin 2x-8)=0$ $(3\sin 2x-2)(\sin 2x+4)=0 \Rightarrow \sin 2x=...$	M1
	$\sin 2x=\dfrac{2}{3}$	A1
	$\left(x=\right)0.365$	A1
	$x=0$	B1
		(4)
		(8 marks)

(a)

M1 Attempts to use $\tan\theta=\dfrac{\sin\theta}{\cos\theta}$ in the given equation. $5\tan\theta=\dfrac{5\sin\theta}{5\cos\theta}$ is M0 **unless** the correct identity is first stated in which case allow for the attempt to use it.

M1 Attempts to multiply both sides by both of their denominators. Condone invisible brackets for the method, but penalise in the A mark.

M1 Uses the identity $\cos^2\theta=1-\sin^2\theta$ to form an equation in $\sin\theta$ only.

A1* Achieves the given answer with no errors including brackets, and no **consistent** incorrect notation (allow if just one instance occurs). Examples of incorrect notation are $\sin\theta^2$ or use of just sin.

(b) Note the question says "hence" so there must be an attempt to use the given, or their, answer from part (a) to score the M.

M1 Attempts to solve the cubic (allow with their cubic if incorrect) by taking out a factor of $\sin 2x$ or cancelling and attempts to solve the quadratic. Usual rules for solving a quadratic. Allow with $2x$ or with θ (or indeed with x or another variable), and allow with $y=\sin 2x$ (oe) used. May be implied by sight of correct value(s) for their cubic/quadratic equation.

A1 $\sin 2x=\dfrac{2}{3}$ or allow for $\sin\theta=\dfrac{2}{3}$, or even $y=\dfrac{2}{3}$ if y has been identified as $\sin 2x$.

(However, do not accept $x=\dfrac{2}{3}$ unless recovered.) Ignore any other solutions (whether incorrect or not).

A1 $\left(x=\right)$ awrt 0.365 (isw after a correct value) and no additional solutions in the range other than 0. Must have gained the M mark. (Degrees answer is A0)

B1 $x=0$ identified as an answer. Must be x May be scored with no working seen. Condone $0°$ and 0.000.

Question Number	Scheme	Marks
7(a)	$(t=10, P=)\quad 1.2$	B1
	$(t=11.5, P=)\quad 2.821$	B1
		(2)
(b)	$h=0.5$	B1
	$E \approx \left(\frac{1}{2}\times"\frac{1}{2}"\right)\left["1.2"+2.95+2(1.882+2.45+"2.821")\right]$	M1 A1ft
	$E \approx 4.61\quad$ (kWh)	A1
		(4)
		(6 marks)

(a)

B1 1.2 or $\frac{6}{5}$. Allow 1.200 May be seen in the table or in their working for part (b).

B1 awrt 2.821 May be seen in the table or in their working for part (b).

(b)

B1 $h=0.5$ (seen or implied)

M1 Scored for the correct $[......]$ bracket structure. It needs to contain their first P value plus the last P value and the inner bracket must be multiplied by 2 and be the summation of the remaining P values in the table with no additional values.

If the only mistake is a copying error or is to omit one value from the inner bracket this may be regarded as a slip and the M mark can be allowed. (An extra repeated term forfeits the M mark however). M0 if values used in brackets are t values instead of P values. Allow recovery of invisible brackets but M0 if never recovered.

A1ft For the correct bracket $[......]$ following through their values in (a).

A1 cao 4.61 Must be to 2 d.p. (note using integration the answer is 4.643439737)

Question Number	Scheme	Marks
8(a)	$a_2 = 2p^2 - 7$	B1
		(1)
(b)	$a_3 = 2("2p^2 - 7" + 3)^2 - 7$	M1
	$p - 3 + 2p^2 - 7 + 8p^4 - 32p^2 + 25 = p + 15$ $\Rightarrow 8p^4 - 30p^2 = 0$	M1 A1
	Correct values of $p^2 = 0$ and $\frac{15}{4}$ (may be implied by $p = 0$ and $\pm\sqrt{\frac{15}{4}}$)	A1
	Uses their values of p to find a_2 using a correct method	dM1
	Possible values for a_2 are -7 and $\frac{1}{2}$	A1
		(6)
Alt (b)	$\sum_{n=1}^{3} a_n = p + 15 \Rightarrow p - 3 + a_2 + 2(a_2 + 3)^2 - 7 = p + 15$	M1
	$\Rightarrow a_2 + 2(a_2 + 3)^2 - 25 = 0$ $\Rightarrow 2{a_2}^2 + 13a_2 - 7 = 0$	M1A1 A1
	$\Rightarrow (2a_2 - 1)(a_2 + 7) = 0 \Rightarrow a_2 = ...$	dM1
	Possible values for a_2 are -7 and $\frac{1}{2}$	A1
		(6)
		(7 marks)

(a)

B1 $(a_2 =)\ 2p^2 - 7$

(b)

M1 An attempt to find a_3 by substituting their a_2 into the formula. Look for $2(...)^2 - 7$ where … is a recognisable attempt at their $a_2 + 3$ (M0 if the −7 is missing).

M1 Attempts to add their first 3 terms and equate to $p + 15$. It is not dependent, and the attempts at the three terms may be poor, as long as they are attempting to add three terms.

A1 $8p^4 - 30p^2 = 0$ oe with terms gathered according to power (need not be all on one side).

A1 $\left(p^2 =\right)\frac{15}{4}$, 0. May be implied by $p = (\pm)\sqrt{\frac{15}{4}}$ and 0. Accept equivalent fractions or decimal answer for $\frac{15}{4}$, or awrt 1.94 for p. May be called by another letter, or even allow if p^2 is mistaken as p.

dM1 Attempts to find at least one value for a_2 using a value for p which has come from an attempt (however poor) at solving their equation. Must be using their formula from part (a). Depends on the second M mark.

A1cso Identifies both -7 and $\frac{1}{2}$ as the possible values for a_2 and no others.

Note that if they have $a_2 = 2p - 7$ by error in (a) they can score M1M1A0A1dM1A0cso in (b).

Alt

M1 Attempts sum of first three terms with a_3 in terms of a_2 and sets equal to $p + 15$

M1 Cancels p's to reach an equation in just a_2

A1 Correct unsimplified equation in a_2

A1 Correct simplified quadratic in a_2

dM1 Solves the quadratic in a_2

A1cso Correct values for a_2

Question Number	Scheme	Marks
9(a) **(i)**	Centre = $(k,2k)$	B1
(ii)	Radius = $\sqrt{k+7}$	B1
		(2)
(b)(i)	$(2-k)^2+(3-2k)^2=k+7 \Rightarrow 4-4k+k^2+9-12k+4k^2=k+7$	M1
	$5k^2-17k+6=0$ *	A1*
(ii)	$(k=)\frac{2}{5},3$	B1
		(3)
(c)		
	Centre is $\left(\frac{2}{5},\frac{4}{5}\right)$	B1ft
	Gradient of tangent $\pm\frac{2-"\frac{2}{5}"}{3-"\frac{4}{5}"}=\left(-\frac{8}{11}\right)$	M1
	$y-3="-\frac{8}{11}"(x-2)\Rightarrow$ sets $y=0\Rightarrow x=...$ Alternatively, $\tan\angle PTO=\frac{8}{11}\Rightarrow XT=\frac{3}{8/11}=...$ where X is (2,0)	M1
	Area $OPT=\frac{1}{2}\times"\frac{49}{8}"\times 3=...$ Alternatively, Area $OPT=\frac{1}{2}\times 2\times 3+\frac{1}{2}\times"\frac{33}{8}"\times 3=...$	dM1
	$=\frac{147}{16}$ oe	A1
		(5)
		(10 marks)

(a)
(i)

B1 $(k,2k)$

(ii)
B1 $\sqrt{k+7}$ and isw after correct answer seen.

(b)
(i)
M1 Substitutes $(2,3)$ into the equation of the circle and attempts to multiply out.

A1* Proceeds to $5k^2 - 17k + 6 = 0$ with no errors

(ii)

B1 $(k =)\frac{2}{5}, 3$ only. Do not isw if they go on to go to give a range of values for *k*.

(c) **NB The method marks may be scored with any *k*.** If a candidate has solved incorrectly and has both values less than 2, they can score for use of either (best attempt) in solving the problem. (If neither value is less than 2 allow the methods for any value of *k* used.)

B1ft Uses the correct value for *k* and finds the centre of the circle. Follow through their $(k, 2k)$ with $k = \frac{2}{5}$. Accept decimal equivalents. Must be clearly identified or used as the centre, not just *k* substituted into the original equation.

M1 Attempts the gradient of the tangent. Look for an attempt at ±(change in *x* / change in *y*) using their centre and *P*. May find the gradient from centre to *P* first and then apply the (negative) reciprocal.

M1 Attempts to find the point *T* by finding the equation of the tangent at *P* using their gradient and $(2,3)$, setting $y = 0$ and proceed to find a value for *x* or equivalent processing. The gradient need not be correct but must be from an attempt with their centre and *P*. Alternatively, the tangent may be used to find angle *PTO* and attempt the length *XT* where *X* is (2,0). (Note they do not need to find *T* in this method.)

dM1 Correct method to find the area of the triangle. It is dependent on the previous method mark. There may be attempts at finding the lengths of each side, the cosine rule to find an angle, and the area formula $\frac{1}{2}ab\sin C$. It must be a complete method with valid attempts to find the appropriate lengths.

A1 $\frac{147}{16}$ oe (decimal is 9.1875 and must be exact)

SC for the dM1

O is not defined in the question and some may use *O* as the centre of the circle. These still need to attempt the first three marks as in the scheme. Also allow the final M mark for a correct attempt at the area *OPT* with *O* as the centre of the circle but must be a complete method. Mostly likely finding the lengths $TP = \frac{3\sqrt{185}}{8}$ and $r = \sqrt{"k"+7}$ (oe) and using $\frac{1}{2}rTP$.

Question Number	Scheme	Marks
10(a)	$d=\dfrac{37-15}{11}\left(=2\right)$	M1
	$u_5=15+4\times"2"=...$	M1
	Week 5 Sunday run = 23 (km)	A1
		(3)
(b)	$r^{11}=\dfrac{37}{15}\Rightarrow r=\sqrt[11]{\dfrac{37}{15}}=...$	M1
	$u_5=15\times"1.0855..."^4=...$	M1
	Week 5 Sunday run = awrt 20.8 (km)	A1
		(3)
(c)	$S_{12}=\dfrac{12}{2}\left[2\times15+(12-1)\times"2"\right]$ or $\dfrac{12}{2}\left(15+37\right)$ $\left(=312\right)$	M1
	$S_{12}=\dfrac{15(1-"1.08554.."^{12})}{1-"1.08554.."}$ $\left(=294.185..\right)$	M1
	Difference = $\left(\dfrac{360-312}{12}\right)$ or $\left(\dfrac{360-294.185...}{12}\right)=...$	dM1
	Either Training model A: $x\leqslant$ 4 km, Training model B: $x\leqslant$ 5.4… km with = or $\leqslant$	A1
	Training model A: x = 4 km, Training model B: x = 5km	A1cso
		(5)
		(11 marks)

(a)

M1 Attempts to find the common difference. Allow for $d = \frac{37-15}{k}$ where k = 11 or 12. Must be using 37 and 15.

M1 Use their common difference to find the distance run on Sunday of week 5. Must be correct formula. They may work out the first 5 terms, in which case look for there being 5 terms.

A1 23 (km)

(b)

M1 A correct method to find r award for $r^{11} = \frac{37}{15}$ oe and proceeding to find a value for r. May be implied by awrt 1.09. Allow if $r^{11} = \frac{37}{15}$ is followed by stating awrt 8.6% increase if no value is given for r.

M1 Uses their value of r to find the distance run on Sunday of week 5. Must be correct formula. They may work out the first 5 terms, in which case look for there being 5 terms.

A1 awrt 20.8 (km)

(c)

M1 Attempts to find the total distance run on Sundays over the 12 weeks using training model A with $n=12$ and **either** $a=15$ and their d or a = 15 and l = 37. The formula must be correct with the values substituted in the correct places. Listing and adding requires all 12 terms

M1 Attempts to find the total distance run on Sundays over the 12 weeks using training model B with $a=15$ and their r. The formula must be correct with the values substituted in the correct places., or listing and adding requires all 12 terms

dM1 Attempts to find the weekly amount run on a Wednesday for **either** model A **or** model B. Look for $\frac{360 - \text{their total}}{12}$. They may set up an inequality in x such as $\text{their total} + 12x \leqslant 360$ and attempt to solve, which is equivalent working.
Allow with =,< instead of $\leqslant$ or with inequalities in the wrong direction.

It is dependent on having scored the previous method mark appropriate for their attempt.

A1 For identifying **either** 4 for model A **or** 5 for model B as the critical values. May be part of an inequality such as $x \leqslant 4$ for model A or $x \leqslant 5.4...$ for model B etc. (Note that use of r = 1.09 will give 4.8… instead of 5.4… and will not be eligible for the A mark if model A is also incorrect.)
Allow with =,< instead of $\leqslant$ or with inequalities in the wrong direction.

A1cso Training model ***A x* = 4** km and training model ***B x* = 5** km
(It must be clear the correct values are linked to the correct models, but they need not be labelled "model A" and "model B")